Transport of Fast Particles in Turbulent Fields



M.J. Pueschel



Max-Planck-Institut für Plasmaphysik

Many thanks to: F. Jenko, T. Hauff, S. Guenter

5th Iter International Summer School, June 20, 2011

Motivation: Fast Particles

Fast particles in the Iter plasma:

- If fusion-born α particles: 3.5 MeV, total power $\sim 100 \,\mathrm{MW}$
- injected neutrals: $1 \, \mathrm{MeV}$, total power $\sim 50 \, \mathrm{MW}$





Tracer Particles vs. Alfvén Eigenmodes

Eigenmodes	Tracers
 fast ions on resonant surfaces 	turbulent thermal species
destabilization of modes	fast ions are deflected
Place No: 51579 Point	
(JET)	(Padberg et al.)

\Rightarrow **both** mechanisms can play an important role in **fast ion transport**

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Experimental Motivation

ASDEX Upgrade discharges with significant NBI heating:



 \Rightarrow need an efficient radial transport mechanism to explain the observed profiles during/after the NBI phase

1 Fast Tracers in Turbulence

- 2 Plasma Microturbulence
- 3 Simulations with Fast Particles

Source & further reading: T. Hauff, Ph.D. thesis, University of Ulm, 2009

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Characteristics of Turbulent Fields

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Structure formation:

Power law spectra:



Turbulence: nondeterministic, need **statistical** approaches

Characteristic quantities

Correlations:

 time: τ_c
 length: λ_c, but generally λ_x ≠ λ_y

Kubo number: (connects turbulence, particle motion)

$$\mathcal{K} = \frac{\tau_c}{\tau_\lambda} = \bar{v} \, \frac{\tau_c}{\lambda_c}$$

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Toroidal Magnetic Geometry

Fusion reactors: toroidal, poloidal magnetic field

tokamak: $B(z) \sim (1 + \epsilon_t \cos z)^{-1}$



shear: $\hat{s} = (R_0/q_0) dq(r)/dr$

0.8 m 0.6 0.4 02 $-2\pi - 1\pi$ 0 1π 2π 7.

stellarator:

Particle Motion in Tori

pitch angle: $\eta \equiv v_{\parallel}/v$

Trapping

- magnetic field: inhomogeneous (∂B/∂z ≠ 0)
- $\eta \to 0$: mirror force $F_{\text{mirr}} \propto \partial f / \partial v_{\parallel}$ traps particle in magnetic well
- banana orbit

Additionally: toroidal precession drift affects both trapped, passing particles



Decorrelation

- short times: ballistic,
 superdiffusive
 movement, before
 turbulent structure is felt
- if a particle is caught by a structure: subdiffusive movement, localized
- only if memory of previous movement (correlation) is lost: diffusive behavior



Diffusivity:

$$D(t) = \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \langle \Delta x^2(t) \rangle$$

Orbit Averaging

Basic idea: if particle orbits move slowly through turbulent structures, orbits can be averaged over

- removes orbit time scale
- particles feel little of averaged-over turbulence
- predicts fast fall-off of diffusivity with particle energy
- turns out to be valid only in **low-shear** regimes of tokamaks



Conditions for validity:

$$T_{
m orbit} \bar{V}_{
m drift} =$$

= $T_{
m orbit} rac{V_{
m drift}}{(2\sqrt{\pi}\Delta x/\lambda_c)^{1/2}} < \lambda_c$

$$T_{\mathrm{orbit}} < \tau_c$$

A Different Approach

If orbit averaging is valid:

- little influence of turbulent eddies on fast particles
- decorrelation through time evolution of turbulence

If orbit averaging is **not** valid ($T_{\text{orbit}}\overline{V}_{\text{drift}} > \lambda_c$):

- particles leave eddie in a single orbit
- T_{orbit} is a good measure for the decorrelation time

Note: alternatively, parallel motion can decorrelate with τ_{\parallel} Typically, however,

$$T_{\text{orbit}} < \tau_{\parallel} < \tau_c$$

Tracers in Electrostatic Fields

For electrostatic turbulence, take $E \times B$ drift velocity v_E , get

$$D_{\rm fi,p}^{\rm es}(E) \approx \frac{v_E^2 \lambda_V}{3\eta^2} \left(\frac{E}{T_{\rm e}}\right)^{-1}$$
$$D_{\rm fi,t}^{\rm es}(E) \approx \frac{1.73 v_E^2 \lambda_c \lambda_V \epsilon_t^{1/2}}{12\pi^{1/2} \eta^2 (1-\eta^2)} \left(\frac{E}{T_{\rm e}}\right)^{-3/2}$$

Passing particles ($\eta \rightarrow 1$): for very large *E*, eventually $D_{\rm fi,p} \propto E^{-3/2}$ when gyroaveraging becomes important

Trapped particles $(\eta \rightarrow 0)$: particles drift farther during one T_{orbit} \Rightarrow orbit averaging breaks down more quickly

Tracers in Magnetic Fields

Magnetic: replace v_E with $v_B = v_{\parallel}B_x/B_0$ (deflection on perturbed field lines: *magnetic flutter*)

$$D_{\rm fi,p}^{\rm em}(E) \approx \left(\frac{B_x}{B_0}\right)^2 \frac{\lambda_B}{3}$$

$$D_{\rm fi,t}^{\rm em}(E) \approx \frac{1.73(B_x/B_0)^2 \lambda_B^2 \epsilon^{1/2} \eta}{12\pi^{1/2}(1-\eta^2)} \left(\frac{E}{T_{\rm e}}\right)^{-1/2}$$

Fluctuation strength

$$\frac{B_x}{B_0} = C \frac{\beta}{\beta_{\text{crit}}} \frac{\rho_s c_s}{R_0}, \quad C \sim \mathcal{O}(0.1 - 1) \quad \text{(from simulations)}$$

 \Rightarrow no or only slow fall-off, at odds with orbit averaging

Magnetic transport: can produce $D_{\rm fi} \sim 0.5 \, {\rm m}^2/{\rm s}$, which is necessary for explaining ASDEX Upgrade results

Runaway Electrons: Motivation

Plasma disruptions

- sudden loss of confinement
- large electric fields
- can accelerate electrons very efficiently

Fast electrons: very low collisionality, $\nu_e \propto v^{-3}$

 \Rightarrow acceleration "without bounds" to $\mathcal{O}(10-1000)\,\mathrm{MeV}$



(Tore Supra)

Previous theories: overestimating radial diffusion of runaway electrons

Runaway Electrons: Results

- relativistic treatment $(E \gg m_e c^2)$
- typically, $\eta \sim 1$
- only D_e^{em} of interest (more efficient than D_e^{es})

Runaway electron diffusivity

$$D_{\rm e}^{\rm em} = \frac{2}{3} \left(\frac{B_x}{B_0}\right)^2 \frac{\lambda_B e B_0 R_0}{m_0} \frac{1}{\gamma_{\rm e}}$$

thus
$$D_{\rm e}^{\rm em} = \propto E^{-1}$$



- at higher energies, $D_{\rm e}^{\rm em} \propto E^{-2}$ (gyroaveraging effects)
- \Rightarrow consistent with observations, but B_x^2 dependence can make quantitative comparisons difficult

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Confinement and Anomalous Transport

Energy confinement time τ_E (as in $nT\tau_E \gtrsim 10^{21} \text{ keV s/m}^3$): governed by radially outward **heat transport**

Transport channels

- collisional (negligible)
- neoclassical (banana orbits)
- radiative
- MHD turbulent (but: equilibrium)
- microturbulent



 \Rightarrow core confinement is limited by Larmor radius scale (kinetic) turbulence

The Gyrokinetic Framework

Kinetic (non-Maxwellian) phenomena generally require **full phase space** description via the Vlasov equation:

3D real space + 3D velocity space + 1D time

Gyrokinetics

To make computations feasible: eliminate fast gyromotion

$$\langle \ldots \rangle = \int\limits_{0}^{2\pi} \ldots \mathrm{d}\theta$$

 \Rightarrow going from 6D to 5D, speed-up by factor $\mathcal{O}(10-100)$



Gyrokinetic Vlasov Equation

$$\begin{split} \frac{\mathrm{d}g_{j}}{\mathrm{d}t} &= 0 = \frac{\partial g_{j}}{\partial t} + \frac{B_{0}}{B_{0\parallel}^{*}} \left[\omega_{n} + \omega_{Tj} \left(v_{\parallel}^{2} + \mu B_{0} - \frac{3}{2} \right) \right] F_{j0} \frac{\partial \chi}{\partial y} - \\ &- \frac{v_{Tj}v_{\parallel}}{JB_{0}} \frac{\partial G_{j}}{\partial z} + \frac{B_{0}}{B_{0\parallel}^{*}} \left(\frac{\partial \chi}{\partial y} \frac{\partial g_{j}}{\partial x} + \frac{\partial \chi}{\partial x} \frac{\partial g_{j}}{\partial y} \right) + \frac{B_{0}}{B_{0\parallel}^{*}} \frac{T_{j0}(2v_{\parallel}^{2} + \mu B_{0})}{q_{j}B_{0}} \cdot \\ &\cdot \left[\mathcal{K}_{x} \frac{\partial G_{j}}{\partial x} + \mathcal{K}_{y} \frac{\partial G_{j}}{\partial y} - \mathcal{K}_{x}F_{j0} \left(\omega_{n} + \omega_{Tj} \left(v_{\parallel}^{2} + \mu B_{0} - \frac{3}{2} \right) \right) \right] - \\ &- \frac{B_{0}}{B_{0\parallel}^{*}} \frac{T_{j0}v_{\parallel}^{2}}{q_{j}B_{0}^{2}} \beta \left(\sum_{j} n_{j0}T_{j0}(\omega_{n} + \omega_{Tj}) \right) \frac{\partial G_{j}}{\partial y} + \frac{v_{Tj}\mu}{2JB_{0}} \frac{\partial B_{0}}{\partial z} \frac{\partial f_{j}}{\partial v_{\parallel}} \end{split}$$

(with the generalized potential $\chi = \bar{\Phi}_j - v_{Tj}v_{\parallel}\bar{A}_{1\parallel j} + (T_{j0}/q_j)\mu\bar{B}_{1\parallel j}$, complemented by the field equations for Φ , $A_{1\parallel}$, $B_{1\parallel}$)

Field Equations

$$\begin{split} A_{1\parallel} &= \left(\sum_{j} \frac{\beta}{2} q_{j} n_{j0} v_{Tj} \pi B_{0} \int v_{\parallel} J_{0}(\lambda_{j}) g_{j} dv_{\parallel} d\mu\right) \times \\ &\times \left(k_{\perp}^{2} + \sum_{j} \frac{\beta q_{j}}{m_{j}} n_{j0} \pi B_{0} \int v_{\parallel}^{2} J_{0}(\lambda_{j}) F_{j0} dv_{\parallel} d\mu\right)^{-1} \\ \Phi &= \left(\sum_{j} q_{j} n_{j0} \pi B_{0} \int J_{0}(\lambda_{j}) g_{j} dv_{\parallel} d\mu\right) \times \\ &\times \left(k_{\perp}^{2} \lambda_{\mathrm{D}}^{2} + \sum_{j} \frac{q_{j}}{T_{j0}} n_{j0} (1 - \Gamma_{0}(b_{j}))\right)^{-1} \end{split}$$

(small plasma pressure β limit; otherwise, need more complicated, coupled Φ - $B_{1\parallel}$ system)

Temperature Gradient Driven Modes

Microturbulence is driven by free energy sources, mostly **temperature** and **density gradients**

ITG modes

- ion temperature gradient driven
- zonal flows



ETG modes

- electron temperature gradient driven
- streamers



Trapped Electron Mode

TEMs

- B₀ inhomogeneity: banana orbits
- $\nabla T_{\rm e}$ or $\nabla n_{\rm e}$ driven
- electron mode on $\sim \rho_i$ scales



Other modes

Microtearing

- large toroidal scales
- restructures magnetic field
- $\nabla T_{\rm e}$ driven

Kinetic Ballooning

- kinetic version of MHD ballooning mode
- strict β limit
- ∇T_i is important
- less well-understood modes . . .

Electromagnetic Effects

Electromagnetic: finite plasma pressure β , leading to magnetic field fluctuations $A_{\parallel} \rightarrow B_x, B_y$

Consequences of high pressure

- essential for high reaction rates
- important for bootstrap fraction, semi-continuous operation of tokamaks
- has complicated impact on microturbulent instabilities (stabilizing/destabilizing)
- eventually leads to loss of confinement at β_{crit} (ballooning threshold)
- responsible for new heat/particle transport channels:
 electromagnetic flutter due to perturbed field

area of ongoing research

Magnetic Transport

Model for electron heat transport along **perturbed field lines** (Rechester & Rosenbluth 1978):

Test particle transport model

$$q_{\mathrm{e\parallel}} = -n_{\mathrm{e0}}\chi_{\mathrm{e\parallel}} \left(\frac{\mathrm{d}T_{\mathrm{e\parallel}}}{\mathrm{d}z} + \frac{B_x}{B_{\mathrm{ref}}}\frac{\mathrm{d}T_{\mathrm{e\parallel}}}{\mathrm{d}x} + \frac{B_x}{B_{\mathrm{ref}}}\frac{\mathrm{d}T_{\mathrm{e0}}}{\mathrm{d}x}\right)$$

Turbulence: third term dominates first, second $\Rightarrow Q_e^{em} \propto \beta^2$ Heat diffusivity:

$$\chi_{\rm e}^{\rm em} = q_0 R \left(\frac{T_{\rm e}}{m_{\rm e}}\right)^{1/2} \langle (B_x/B_{\rm ref})^2 \rangle$$

 \Rightarrow magnetic transport can become comparable with electrostatic transport at reactor-relevant β

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Numerical Approach

Solving gyrokinetic Vlasov equation requires advanced code: Gyrokinetic Electromagnetic Numerical Experiment

The GENE Code

- runs on up to 10⁵ cores
- initial/eigenvalue code
- advanced collision operator
- high-β physics
- tokamaks and stellarators
- worldwide user base
- available to researchers: gene.rzg.mpg.de



GENE Diagnostics Tool

- post-processing suite
- standard analyses, e.g.: spectra/profiles/contours
- advanced diagnostics:
 3D rendering textures, field line integrator

Passive Species Simulations

Active (a) vs. **passive** (p) particle species

Vlasov equation used for all species

$$\frac{\partial g_a}{\partial t} = f(a, \Phi, A_{1\parallel})$$
 and $\frac{\partial}{\partial t}$

and
$$\frac{-t}{\partial t} = j$$

$$\frac{\partial g_p}{\partial t} = f(p, \Phi, A_{1\parallel})$$

while the field equations

$$\Phi\propto\sum_{a}\int g_{a}\mathrm{d} v_{\parallel}\mathrm{d} \mu$$
 and $A_{1\parallel}\propto\sum_{a}\int g_{a}\mathrm{d} v_{\parallel}\mathrm{d} \mu$

only depend on a

Here: passive fast ion species with $T_{\rm fi} = 30 - 100T_{\rm e}$ \Rightarrow significant impact on numerical time step!

Note: fi species has Maxwellian distribution, but analyses divide by F_{f0}

Simulation Results



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General Geometry

Previous results: simple \hat{s} - α geometry (circular flux surfaces), but realistic equilibria **may affect** fast particle **diffusion** Experimental results available for ASDEX Upgrade, DIII-D \Rightarrow take respective **discharge geometries**



Note: shaping affects fast ions also via turbulence (e.g.: λ_c , B_x)

work in progress ...

ASDEX Upgrade



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DIII-D



electrostatic-magnetic intersection at low energies:

magnetic component important

General geometry conclusions (*preliminary*!)

- *D*_{em} not negligible
- Shaping increases influence of *D*_{em}
- impact of η needs to be considered

Simulation Database

GENE database (ASDEX Upgrade, DIII-D, \hat{s} - α):



 \Rightarrow can use simple equations with experimentally available quantities:

$$\frac{2/3 v_E^2}{40 \left(B_x/(\beta/\beta_{\rm crit})\right)^2} \approx \chi_{\rm total}^{\rm es}$$

Additionally: avoid λ s by using passive particle gauge

Summary I

- Fast particles: susceptible to background turbulence
- Recent ASDEX Upgrade experiments: significant radial diffusion
- Standard model Orbit Averaging cannot explain findings
- Decorrelation occurs through drift motion over a *single* orbit
- Magnetic diffusion of passing fast ions: no E dependence, providing an efficient transport mechanism
- Runaway electrons: previous theories cannot explain suppressed diffusion relativistic treatment shows: $D \propto E^{-2}$ at large energies (gyroaveraging)

Summary II

Background species: gradients drive turbulence on the scale of gyroradii—ITG, TEM, ETG, KBM, MT, …

Simulations:

- self-consistent turbulence from active species
- passive species evolve according to gyrokinetic Vlasov equation
- Results show excellent agreement with theory, in particular no energy dependence of D^{em}_{fi.p}

Outlook

- Approach has already been used successfully to explain astrophysical phenomena (cosmic rays)
- Building up database through simulation evaluation: convenient access to quantities required to obtain diffusivities, using only *experimentally available* data
- Impact of plasma shaping, pitch angle: work in progress